Common Core Standards:
7.RP.1, 7.RP.3

Complex Fractions

Getting the Idea

To divide fractions, first find the **reciprocal** of the divisor. Then multiply the dividend by the reciprocal of the divisor. Reciprocals are two numbers whose product is 1.

You can find the reciprocal of a fraction or whole number by switching the numerator and the denominator. For example, $\frac{3}{8}$ and $\frac{8}{3}$ are reciprocals because $\frac{3}{8} \times \frac{8}{3} = \frac{3 \times 8}{8 \times 3} = \frac{24}{24} = 1$.

Example 1

Divide.

$\frac{3}{5} \div \frac{2}{3} = \square$

Strategy **Multiply the dividend by the reciprocal of the divisor.**

Step 1 Rewrite as a multiplication problem, using the reciprocal of the divisor.

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

$\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2}$

Step 2 Multiply.

$\frac{3}{5} \times \frac{3}{2} = \frac{3 \times 3}{5 \times 2} = \frac{9}{10}$

$\frac{9}{10}$ is in simplest form.

Solution $\mathbf{\frac{3}{5} \div \frac{2}{3} = \frac{9}{10}}$

A **complex fraction** is a fraction in which the numerator and/or denominator contains a fraction. Recall that a fraction represents a quotient. The quotient is the numerator divided by the denominator (where the denominator is not equal to 0). For example, $\frac{3}{4} = 3 \div 4$.

The division expression in Example 1 can be written as a complex fraction: $\frac{\frac{3}{5}}{\frac{2}{3}}$.

The numerator of the complex fraction is $\frac{3}{5}$ and the denominator is $\frac{2}{3}$.

The complex fraction and the division expression are equivalent: $\frac{\frac{3}{5}}{\frac{2}{3}} = \frac{3}{5} \div \frac{2}{3}$.

You can express a percentage as a complex fraction and vice versa.

Example 2

Express 4.25% as a complex fraction.

Strategy **Use the definition of percent to write the complex fraction.**

Step 1 Convert the percent to a fraction.

Percent means per hundred.

Divide the percentage by 100 and drop the percent sign.

$4.25\% \longrightarrow \frac{4.25}{100}$

Step 2 Convert the decimal to an improper fraction.

Since $0.25 = \frac{25}{100} = \frac{1}{4}$, $4.25 = 4\frac{1}{4}$.

$4\frac{1}{4} \longrightarrow \frac{(4 \times 4) + 1}{4} = \frac{17}{4}$

Step 3 Write the complex fraction.

Write the improper fraction over 100.

$$\frac{\frac{17}{4}}{100}$$

Solution **4.25% written as a complex fraction is $\frac{\frac{17}{4}}{100}$.**

Simplifying a complex fraction is the same as dividing its numerator by its denominator.

Example 3

Simplify.

$$\frac{\frac{1}{4}}{\frac{1}{12}} = \square$$

Strategy **Multiply the numerator by the reciprocal of the denominator.**

Step 1 Rewrite as a multiplication problem using the reciprocal of the denominator.

The reciprocal of $\frac{1}{12}$ is $\frac{12}{1}$.

$\frac{1}{4} \div \frac{1}{12} = \frac{1}{4} \times \frac{12}{1}$

Step 2 Multiply.

$\frac{1}{4} \times \frac{12}{1} = \frac{1 \times 12}{4 \times 1} = \frac{12}{4}$

Step 3 Write the answer in simplest form.

$\frac{12}{4} = \frac{12 \div 4}{4 \div 4} = \frac{3}{1} = 3$

Solution $\frac{\frac{1}{4}}{\frac{1}{12}} = 3$

To divide mixed numbers, first rewrite the mixed numbers as improper fractions. Then follow the rules for dividing fractions.

Example 4

Jamie divided $5\frac{1}{4}$ pounds of apples into baskets that hold $1\frac{3}{4}$ pounds each.

How many baskets did she use?

Strategy **Rewrite the mixed numbers as improper fractions. Then divide.**

Step 1 Write an expression to represent the problem.

Find $5\frac{1}{4} \div 1\frac{3}{4}$.

Step 2 Rewrite the mixed numbers as improper fractions.

$$5\frac{1}{4} \longrightarrow \frac{(5 \times 4) + 1}{4} = \frac{21}{4}$$

$$1\frac{3}{4} \longrightarrow \frac{(1 \times 4) + 3}{4} = \frac{7}{4}$$

$$5\frac{1}{4} \div 1\frac{3}{4} = \frac{21}{4} \div \frac{7}{4}$$

Step 3 Rewrite as a multiplication problem using the reciprocal of the divisor.

The reciprocal of $\frac{7}{4}$ is $\frac{4}{7}$.

$$\frac{21}{4} \div \frac{7}{4} = \frac{21}{4} \times \frac{4}{7}$$

Step 4 Simplify the factors and multiply.

$$\frac{\overset{3}{\cancel{21}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{7}}} = \frac{3 \times 1}{1 \times 1} = \frac{3}{1}$$

Step 5 Simplify.

$$\frac{3}{1} = 3$$

Solution **Jamie used 3 baskets.**

Any whole number can be expressed as a fraction. For example, $4 = \frac{4}{1}$. So, the reciprocal of a whole number divisor is a unit fraction. For example, the reciprocal of 4 is $\frac{1}{4}$.

Example 5

Divide.

$6\frac{5}{8} \div 3 = \square$

Strategy **Rewrite the whole number as a fraction. Then find the reciprocal.**

Step 1 Rewrite $6\frac{5}{8}$ as an improper fraction. Write the reciprocal of 3.

$6\frac{5}{8} \rightarrow \frac{(6 \times 8) + 5}{8} = \frac{53}{8}$

The reciprocal of 3 is $\frac{1}{3}$.

Step 2 Rewrite as a multiplication problem and solve.

$6\frac{5}{8} \div 3 = \frac{53}{8} \times \frac{1}{3}$

Step 3 Multiply.

$\frac{53}{8} \times \frac{1}{3} = \frac{53 \times 1}{8 \times 3} = \frac{53}{24}$

Step 4 Simplify the product.

$\frac{53}{24} = 2\frac{5}{24}$

Solution $\mathbf{6\frac{5}{8} \div 3 = 2\frac{5}{24}}$

Coached Example

Mr. Camara cuts a 15-foot wooden board into pieces that are each $1\frac{2}{3}$ feet long. How many pieces of wood does he have?

Let *w* represent the number of pieces of wood.

Write a number sentence to represent this problem. _____

Rewrite 15 as an improper fraction. _____

Rewrite $1\frac{2}{3}$ as an improper fraction. _____

Rewrite the number sentence using improper fractions. _____

To divide fractions, multiply the dividend by the _____ of the divisor.

The reciprocal of the divisor is _____.

Rewrite as a multiplication problem using the reciprocal of the divisor.

_____ ÷ _____ = _____ × _____

Multiply.

_____

Simplify the product. _____

Mr. Camara has _____ pieces of wood.

Lesson Practice

Choose the correct answer.

1. Divide.

$$\frac{1}{3} \div \frac{1}{8} = \square$$

A. $\frac{1}{24}$

B. $\frac{3}{8}$

C. $2\frac{2}{3}$

D. 24

2. Mrs. Chapman made vests for cast members of the school play. She used $\frac{3}{4}$ yard of material for each vest. She used 6 yards in all. How many vests did she make?

A. 1

B. 4

C. 6

D. 8

3. Divide.

$$\frac{7}{16} \div 2\frac{3}{8} = \square$$

A. $\frac{7}{46}$

B. $\frac{7}{38}$

C. $1\frac{5}{128}$

D. $2\frac{4}{7}$

4. Which complex fraction is equivalent to 8.15%?

A. $\frac{815}{100}$

B. $\frac{\frac{15}{8}}{100}$

C. $\frac{\frac{163}{20}}{100}$

D. $\frac{\frac{8}{15}}{100}$

5. Kelly had a ribbon that was $5\frac{1}{3}$ feet long. Each piece she cut was $1\frac{1}{3}$ feet long. How many pieces of ribbon did she cut?

A. 1

B. 4

C. 8

D. 12

6. Divide.

$$2\frac{1}{5} \div \frac{1}{10} = \square$$

A. 11

B. 21

C. 22

D. 221

7. What is the value of the following expression?

$$\frac{1}{2} \div \frac{1}{12}$$

A. 3

B. 6

C. 9

D. 24

8. Which quotient is less than 1?

A. $\frac{5}{8} \div \frac{2}{3}$

B. $\frac{7}{10} \div \frac{3}{5}$

C. $\frac{3}{4} \div \frac{1}{3}$

D. $\frac{7}{8} \div \frac{7}{9}$

9. In math class, Ms. Kuramoto wrote the following complex fraction on the board: $\frac{\frac{5}{6}}{\frac{4}{9}}$.

A. Rewrite the complex fraction as a division expression.

B. Show the complex fraction $\frac{\frac{5}{6}}{\frac{4}{9}}$ in simplest form. Show your work.

10. Look at each equation. Is the equation true? Select Yes or No.

A. $\frac{2}{3} \div \frac{1}{9} = 6$ ○ Yes ○ No

B. $2\frac{1}{10} \div \frac{3}{10} = 7$ ○ Yes ○ No

C. $3\frac{3}{5} \div \frac{4}{5} = 5\frac{1}{4}$ ○ Yes ○ No

D. $\frac{7}{9} \div \frac{1}{3} = \frac{3}{7}$ ○ Yes ○ No

11. Select True or False for each equation.

A. $\frac{9}{16} \div \frac{1}{4} = 2\frac{1}{4}$ ○ True ○ False

B. $2\frac{1}{2} \div 1\frac{2}{3} = 1\frac{1}{2}$ ○ True ○ False

C. $\frac{9}{10} \div \frac{1}{6} = \frac{3}{20}$ ○ True ○ False

D. $\frac{7}{8} \div 1\frac{1}{2} = \frac{7}{12}$ ○ True ○ False

12. Which complex fraction is equivalent to 9.25%? Circle all that apply.

A. $\frac{\frac{925}{10}}{100}$

B. $\frac{\frac{37}{100}}{4}$

C. $\frac{\frac{37}{4}}{100}$

D. $\frac{\frac{9}{25}}{100}$

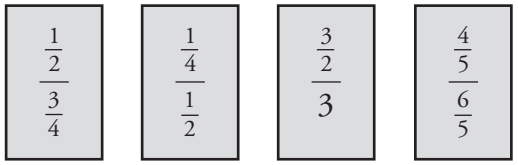
E. $\frac{925}{\frac{200}{2}}$

F. $\frac{925}{\frac{40{,}000}{4}}$

13. Emanuel had $2\frac{1}{4}$ gallons of paint. He used $\frac{3}{4}$ gallon of paint for each room. Circle the number of rooms that Emanuel painted.

Emanuel painted [2 | 3 | 4 | 6] rooms.

14. Simplify each complex fraction. Write each complex fraction in the correct box.



Equal to $\frac{2}{3}$	Equal to $\frac{1}{2}$

15. Use numbers from the box to complete each equation.

$\frac{7}{8} \div$ _____ $= 3\frac{1}{2}$

$\frac{1}{16} \div \frac{5}{8} =$ _____

$2\frac{2}{5} \div$ _____ $= 4\frac{4}{5}$

$\frac{1}{3} \div \frac{2}{3} =$ _____

$4\frac{1}{2} \div$ _____ $= 45$

$\frac{1}{10}$ $\frac{1}{4}$ $\frac{1}{2}$